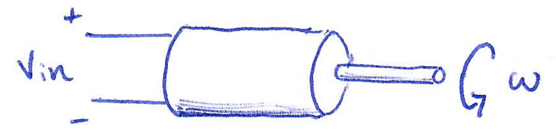
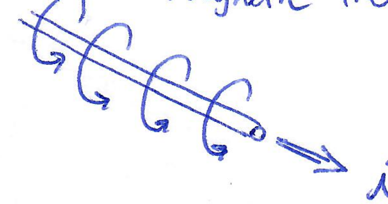
ME 4555 - Lecture 7 - DC motors



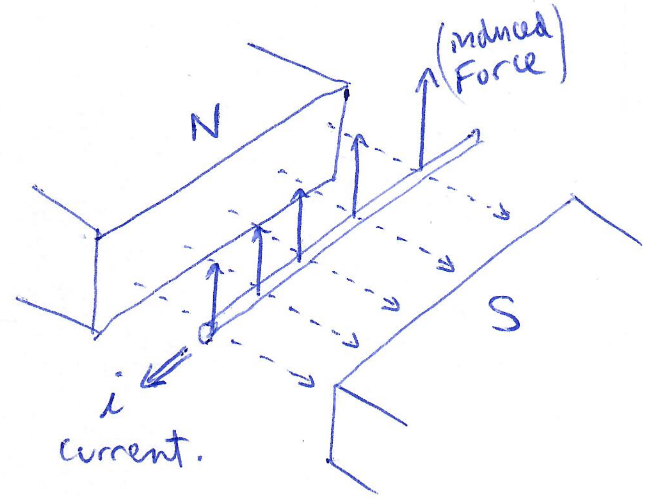
Voltage supply causes shaft to spin ($\omega = \dot{\theta}$ is angular speed of shaft)

Three fundamental principles (Lorentz's force law + Faraday's laws of induction):

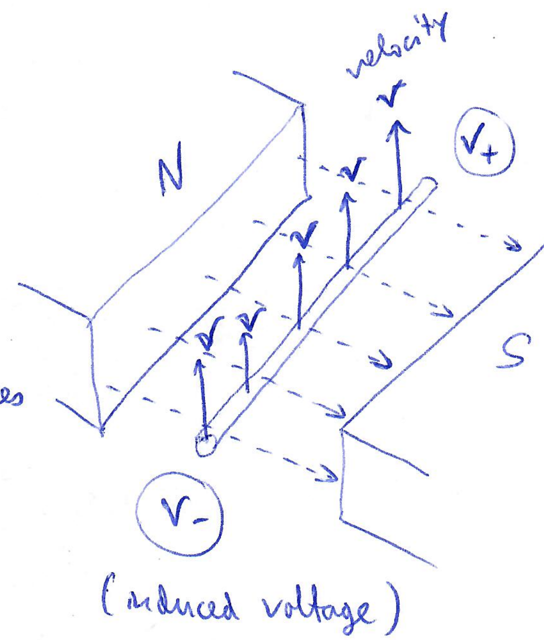
1) A current in a wire establishes a magnetic field around the wire:



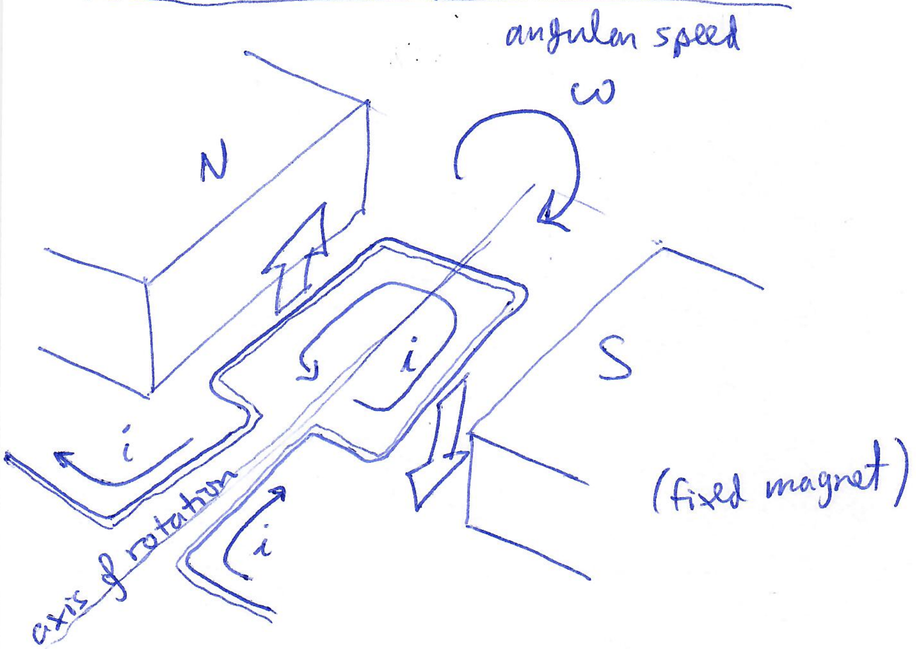
2) A current-carrying wire in a magnetic field has a force on it (perpendicular to the wire + field) and proportional to current: $F \propto i$



3) A wire moving in a magnetic field will induce a voltage between the ends of the wire. The wire becomes like a voltage source whose voltage opposes the current from step 2 that might cause a force that would produce this velocity in the wire.

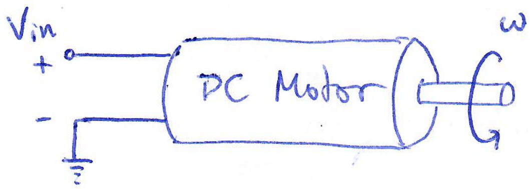


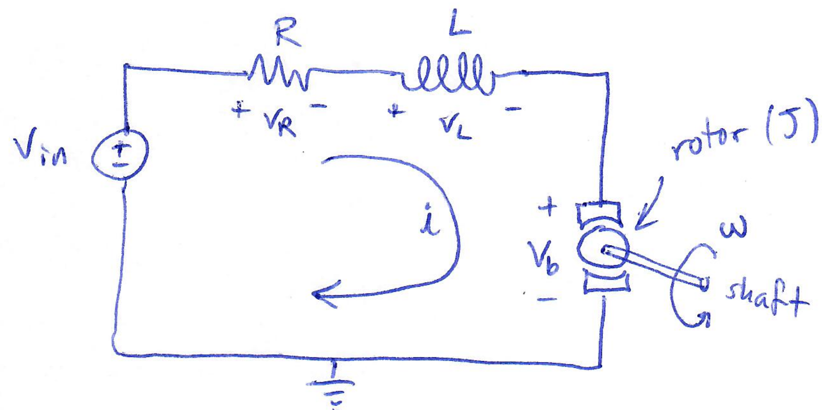
Basic idea of a motor:



- A loop of wire sits in a fixed magnetic field (from fixed magnet)
- Current pushed through wire causes forces on the loop, makes it spin.

Model circuit:





R models the loss in the long wire (resistive / heat losses)

L models the coil effect acting like an inductor.

V_b is the voltage produced by the ~~motor~~ spinning that opposes the current. Also called "back emf".

CAUSAL CHAIN:

★ From law #2, current induces a force (torque) in the shaft.

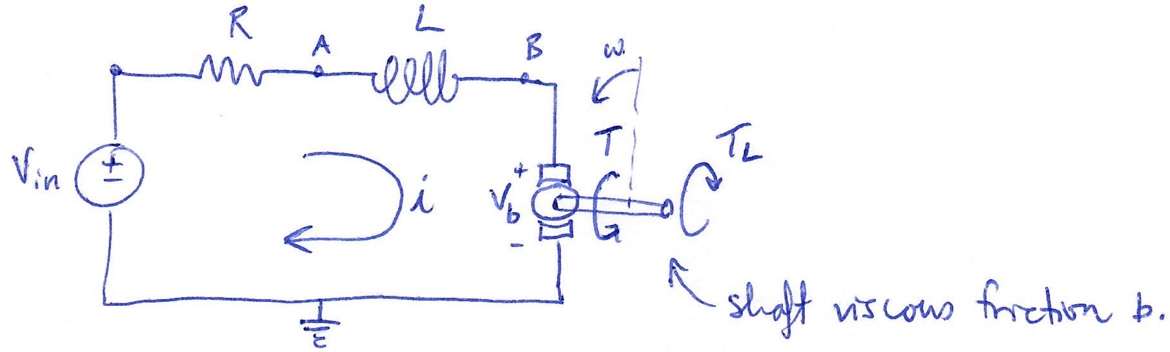
↓

★ From mechanics EOM, torque in shaft induces rotation (Newton's 2nd law).

↓

★ From rotation motion (law #3), voltage V_b is induced that opposes original current.

{ this continues until balance is achieved?



Electrical part:

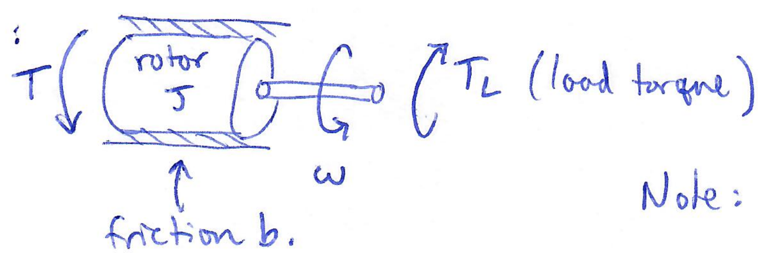
$V_{in} \xrightarrow[i]{R} V_A$ $\qquad V_{in} - V_A = iR$

$V_A \xrightarrow[i]{L} V_B$ $\qquad V_A - V_B = L\dfrac{di}{dt}.$

KVL: $\boxed{L\dfrac{di}{dt} + Ri + V_b = V_{in}}$

Electrical → mechanical: $\boxed{T = K_m i}$ where K_m is the "motor-torque constant" (supplied by the motor manufacturer). Units of K_m are $\left[\dfrac{N\cdot m}{A}\right]$.

Mechanical part:



Note: $\dot{\theta} = \omega$.

EOM: $\boxed{J\dot{\omega} = T - \underbrace{T_L}_{\text{load}} - \underbrace{b\omega}_{\text{friction}}}$

mechanical → electrical: $\boxed{V_b = K_b \cdot \omega}$ where K_b is the "back-emf constant" (supplied by the manufacturer) Units of K_m are $\left[\dfrac{\text{Volts}}{\text{rad/sec}}\right]$ or more commonly $\left[\dfrac{\text{Volts}}{\text{rpm}}\right]$.

Note: If using SI units, $K_m = K_b$. This is because power is conserved. So (mechanical power) = (electrical power)

$$\Rightarrow \quad P = \underbrace{T\omega}_{\text{mechanical}} = \underbrace{V_b\, i}_{\text{electrical.}}$$

Note: must use SI units! i.e. $\left[\frac{N\cdot m}{A}\right]$ for K_m and $\left[\frac{\text{Volt}\cdot\text{sec}}{\text{rad}}\right]$ for K_b

But $T = K_m i$ and $V_b = K_b \omega$.

$$\text{so} \quad P = K_m i\omega = K_b i\omega \quad \Rightarrow \quad \boxed{K_m = K_b}$$

All equations for the motor:

$$\left\{\begin{array}{l} L\frac{di}{dt} + Ri + V_b = V_{in} \\ T = K_m i \\ J\dot{\omega} = T - T_L - b\omega \\ V_b = K_b\omega \end{array}\right\} \Longrightarrow \boxed{\left\{\begin{array}{l} L\frac{di}{dt} + Ri + K_b\omega = V_{in} \\ J\dot{\omega} + b\omega - K_m i = -T_L \end{array}\right.}$$

It's possible to eliminate i and get a single equation for ω.

Solve for i in 2nd eqn: $i = \left(\frac{J\dot{\omega} + b\omega + T_L}{K_m}\right)$.

Substitute into 1st eqn:

$$L\left(\frac{J\ddot{\omega} + b\dot{\omega} + \dot{T}_L}{K_m}\right) + R\left(\frac{J\dot{\omega} + b\omega + T_L}{K_m}\right) + K_b\omega = V_{in}$$

$$\Rightarrow \boxed{\frac{LJ}{K_m}\ddot{\omega} + \left(\frac{RJ + Lb}{K_m}\right)\dot{\omega} + \left(\frac{Rb}{K_m} + K_b\right)\omega = \underbrace{\left(V_{in} - \frac{L}{K_m}\dot{T}_L - \frac{R}{K_m}T_L\right)}_{\substack{\text{external inputs} \\ \text{(Torques, voltages)}}}}$$

For an ideal motor, we often assume $L = 0$, $b = 0$ (frictionless)

$$\Rightarrow \boxed{\frac{RJ}{K_m}\dot{\omega} + K_b\omega = V_{in} - \frac{R}{K_m}T_L}$$

Chain of causality for DC motors.

Initially, $\omega = 0$ (no motion), no voltage or current.

- connect the battery, i.e. $V_{in} > 0$.
- V_{in} causes current to flow through the windings.
- Flowing current generates torque on shaft ($T = K_m i$)
- Torque on shaft causes acceleration due to $\sum T_{ext} = J\dot{\omega}$
- ω (angular speed) increases as direct result of acceleration.
- motion of coil causes back-emf: $V_b = K_b \omega$.
- V_b causes reduction in current entering the coil

Ultimately, balance is achieved and the motor spins at a constant speed. To find this speed, set $\dot{\omega} = \ddot{\omega} = 0$ in the differential equation:

$$\left(\frac{Rb}{K_m} + K_b\right)\omega = \left(V_{in} - \frac{R}{K_m}T_L\right)$$

$$\Rightarrow \omega = \left(\frac{V_{in} - \frac{R}{K_m}T_L}{\frac{Rb}{K_m} + K_b}\right)$$

Intuition:

larger V_{in} $\rightarrow$ faster ω.

larger T_L $\rightarrow$ slower ω.

larger b $\rightarrow$ slower ω

Interesting:

★ J doesn't matter! Affects rate of change of speed $\dot{\omega}$ (acceleration) but not final speed.

★ Same with L; doesn't affect final speed.

Bonus: motors in reverse: generators!

motor:

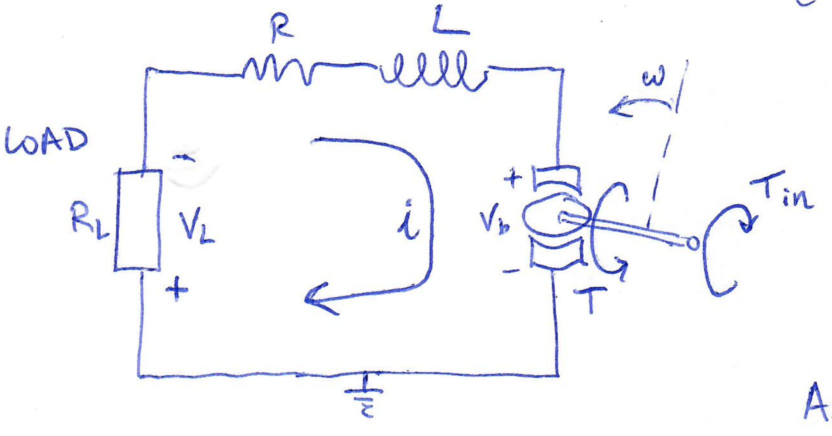
Voltage input
↓
→ current in windings
↓
torque on shaft
↓
shaft spins
↓
back emf (reduces current)

generator:

Torque input on shaft
↓
→ shaft spins
↓
back emf
↓
current in windings (and through load)
↓
torque induced on shaft (opposes input torque)

The equations for a generator are the same as for a motor!

{let's keep the same sign conventions}.



same equations as for motor:

$$\begin{cases} L\dfrac{di}{dt} + Ri + K_b\omega = -V_L \\ J\dot{\omega} + b\omega - K_m i = -T_{in} \end{cases}$$

Assume load is R_L. So $V_L = iR_L$.

For a generator, we typically care about the current driven through the load. So we can eliminate ω instead. After some algebra...

$$\left(\frac{JL}{K_b}\right)\frac{d^2i}{dt^2} + \left(\frac{J(R+R_L)+Lb}{K_b}\right)\frac{di}{dt} + \left(\frac{b(R+R_L)}{K_b} + K_m\right)i = T_{in}$$

So steady-state current through load is:

$$\boxed{i_{ss} = \frac{T_{in}}{\dfrac{b(R+R_L)}{K_b} + K_m}}$$